BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 63 (67), Numărul 1, 2017
Secţia
CONSTRUCŢII DE MAŞINI

INSTANTANEOUS ANGULAR SPEED MEASUREMENT AND SIGNAL PROCESSING: A BRIEF REVIEW

BY

IONUȚ CIURDEA and MIHĂIȚĂ HORODINCĂ*

"Gheorghe Asachi" Technical University of Iaşi,
Faculty of Machine Manufacturing and Industrial Management

Received: February 10, 2017
Accepted for publication: November 9, 2017

Abstract. The paper proposes a brief review of the major available instantaneous angular speed (IAS) measurement and signal processing techniques, useful in various fields of scientific research and industrial applications. A short review on some available resources and results in the literature was also done in order to identify some future objectives in scientific research. The main techniques related by direct and indirect measurement was firstly introduced (discrete-time, continuous-time and encoder-less signal techniques). Some considerations on available IAS signal production techniques (elapsed time measurement, angular displacement numerical differentiation) and processing techniques (numerical filtering, numerical fast Fourier transform) were done, frequently using results from our preliminary research as examples.

Keywords: rotational motion; angular speed; sensor; measurement; signal processing.

1. Introduction

The main reason of this paper is related with an observation on the properties of the components of mechanical power flow (torque and angular

*Corresponding author; *e-mail*: horodinca@tuiasi.ro

speed) transmitted inside a rotational kinematic chain: there is a relationship between instantaneous values of torque and angular speed. This speed-torque relationship (usually with negative slope: increasing the load means decreasing the angular speed) is given by the properties of actuation systems (*e.g*. the mechanical characteristic of any electric motor).

A natural consequence occurs here: the mechanical loading in a kinematic chain (torque) can be evaluated by measuring the instantaneous angular speed (IAS) evolution. This consequence implies the possibility of indirectly measurement of the continuous and variable parts of torque, already revealed by (Aono *et al*., 2013; Villanueva *et al*., 2011).

The measurement of torque supposes to use loading sensors placed on a shaft (usually these sensors acquire the angular deformation of a shaft placed in series on the power flow). The IAS measurement can be done easier, usually with contactless methods, with the sensor placed -temporary if it is necessary- in parallel with the power flow.

In the literature there are many previous researches and results related by computer assisted IAS measurement and data processing in time and especially in frequency domain (related by torsional vibrations). This last topic (IAS analysis in frequency domain) seems to be very promising according with some already published noticeable results as it follows:

– the paper (Li and Zhang, 2017) introduce new monitoring techniques for IAS measurement named Instantaneous Angular Phase Demodulation (IAPD) useful in gearboxes condition monitoring. With the same objectives (Li *et al*., 2017) proposes a new technique of IAS signal processing based on Empirical Mode Decomposition and Autocorrelation Local Cepstrum techniques.

– the paper (Stander and Heyns, 2005) present a new data processing method based on IAS and vibration signals monitoring in order to detect the faults in gears working under cyclic stationary and non-cyclic stationary load fluctuation;

– the papers (Gomez *et al*., 2016; Renaudin *et al*., 2010) propose the IAS measurement as a technique of monitoring for bearing condition.

– the paper (Lamraoui *et al*., 2014) proposes the using of IAS signal for chatter detection in milling.

– the paper (Girardin *et al*., 2010) proposes the using of IAS signal for milling tool condition detection (cutting tool wear and breaks).

In order to achieve a correct approach for a future research activity, this paper proposes to investigate the state-of-the-art on IAS signal measurement and analysis. There are many papers focused on achievements on this subject. It is expected that this study revealed some important resources for our future research which can be exploited later on.

2. Measurement Techniques

2.1. IAS Measurement by Discrete-time Signal Techniques

As general technique for IAS measurement the scientists proposes the conversion of information related by rotation frequency in IAS. An IAS sensor consisting of a rotary part (placed on a shaft whose speed should be measured) which activates (optically, electrically or through a magnetic field) a fixed part (no contact between) is generally used. Some techniques related by special applications use atypical devices.

2.1.1. *IAS Measurement Using Encoders*

Generally the information related by IAS is generated by an encoder placed on the body whose rotation must be monitored. Usually the encoder it's an optical device with a disk inside which contains *N* equidistant angular markers (with a constant angle $\Delta\varphi = 2\pi/N$ between). The encoder produce a number *N* of electrical pulses at each completely revolution of the internal disk. The time delay Δt between two subsequent pulses and the number *N* are involved in calculus of average IAS as follows:

$$\overline{\omega} = \frac{\Delta\varphi}{\Delta t} = \frac{2\cdot\pi}{N\cdot\Delta t} \tag{1}$$

This equation replaces the exact definition of angular speed: $\omega = d\varphi/dt$, with $d\varphi$ and dt infinitely small increments. For a rotating object having a known evolution of rotation angle $\varphi(t)$ (experimentally acquired with a sensor) it is possible the calculus of IAS evolution $\omega(t)$ by numerical differentiation. This can be a privileged direction of research in the future, even if the numerical differentiation seems to increase the measurement noise.

The resolution of $\overline{\omega}$ description using an encoder is *N* values per rotation. Assuming that the encoder shaft rotates with *n* rotation per second then the sampling ratio *R* of $\overline{\omega}$ description is $R=n\cdot N$ samples per second (s^{-1}). The same sampling ratio of $\overline{\omega}$ is obtained if *N* has a high value and the encoder rotates of low speed or *N* has a low value and the encoder rotates at high speed.

This measurement technique (having a big advantage: it does not need calibration) was successfully used by:

– Bourdon *et al.* (2014) which uses a high accuracy optical encoder (with *N* = 5000) for monitoring IAS on rotating machines, in order to detect faults in gears and bearings.

– Leclère *et al.* (2013) which exposes an overview of different types of IAS measurement errors using encoders, in theoretical and experimental terms.

– Diamond *et al*. (2016) which proposes a technique of compensation for geometrical errors of encoders.

– Bourogaoui *et al*. (2016), which make a deep review of different measurement techniques, some issues are related by IAS measurement with encoders.

– André *et al*. (2014) which proposes a method for encoder signal processing taking into account the influence of aliasing and discretization errors on IAS measurement.

– Zhao *et al*. (2018) proposes the Kurtosis-guided local polynomial differentiator method in IAS description useful in fault signatures of defective components on planetary gearboxes.

– Gubran and Sinha (2014) which uses the IAS evolution to detect torsional vibrations induced by transverse blade vibrations in rotating machines.

– Gu *et al*. (2006) which proposes a technique of IAS measurement in presence of electric noise in order to detect faults in rotating parts of a machine.

– Stander and Heyns (2005) which uses an encoder and an accelerometer to generate the IAS and vibrations inputs signals in an online vibration-monitoring systems of gearboxes under non-cyclic stationary load conditions.

Because there is no physical (mechanical) contact between the fixed and the rotating parts of the encoders, this measurement technique doesn't introduce supplementary mechanical friction torque in the supervised system. However a very small dynamic loading occurs (because the inertia of the disk).

2.1.2. *IAS Measurement Using Devices Similarly to Encoders*

Some other measurement techniques use a very similar principle: a proximity sensor activated by a structure placed on a rotational shaft. Some examples are provided here below:

– Rivola and Troncossi (2014) uses a zebra tape with black and white stripes placed on a camshaft. The tape is tracked by an optical probe.

– Espadafor *et al*. (2014) uses a precisely poled band (with 180 teeth) strapped on the engine shaft and a proximity inductive sensor.

– Yu and Zhang (2010) uses a magnetic encoder mounted on a rotating shaft in order to measure the IAS and acceleration of the crankshaft in an aircraft engine-propeller.

– Kim and Park (2007) proposes IAS measurement on a fuel-injection engine using a permanent magnet around which a coil is wound, and it is placed near a flywheel with 100 teeth (N=100). In fact this is a so-called variable reluctance sensor acting as an AC voltage generator (or as AC tachometer). During the rotation of the flywheel this sensor introduces a variable torque, not suitable for many other applications.

– Guo *et al*. (2017) uses a similar technique in a close loop feedback system related to speed in order to reduce the torsional vibrations in a diesel engine shaft.

– Pan *et al*. (2016) uses a three phases Hall position sensor placed inside a DC motor in order to generate an IAS signal useful for regenerative braking in electric vehicles.

Some uncommon measurement techniques use rotational position sensors, with a relative difficult IAS signal constitution procedure:

– Sarma *et al*. (2008) and Bourogaoui *et al.* (2016) propose an angular position and speed sensing technique based on resolvers in servo controlled systems.

– Bellini and Bifaretti (2006) proposes a method of reduction of the electrical noise -by filtering- in the IAS signal generated from an electromagnetic resolver by numerical derivation.

– Wu *et al*. (2017) introduce a measurement technique of rotation angle (with 0.1^{o} resolution) based on a multi-polar magnetic ring placed on a rotary shaft and a modulation coil with a magnetostrictive/piezoelectric laminate composite beam inside as sensor. The IAS information comes either from numerical derivation of the angle either from the frequency of the signal delivered by sensor.

Some IAS measurement techniques use AC tachometers (Altintas *et al*., 2011; Shimano *et al*., 1990). These tachometers usually generate a harmonic signal. The IAS signal is related by frequency or amplitude. In the last case, the IAS and amplitude are directly proportional; the proportionality constant should be determined by calibration. Some of our researches proves that a stepper motor (used as generator) works properly also as an AC tachometer.

In the major part of these techniques the IAS information occurs as a discrete-time signal, generally not suitable for real-time close-loop feedback control with a simple regulator. The time-delay in IAS signal processing can generate dynamic instability.

2.2. IAS Measurement by Continuous-time Signal Techniques

An easy way to describe the IAS evolution is to use an inexpensive analog DC tachometer (or a brushes DC motor used as generator) which provides a DC voltage *u* proportional with the instantaneous angular speed:

$$u = K \cdot \omega \tag{2}$$

Compared to the major part of the techniques outlined above, the IAS measurement with a DC tachometer provides an important advantage: it generates an electric signal directly related by IAS, no need external supplementary electrically powered devices.

Unfortunately there are some disadvantages of this measurement technique: it needs calibration and signal processing (low pass filtering) in order to remove the important electrical noise (generated by the brushes used to collect the signal from the tachometer rotor). Unfortunately the filtering removes essential information of the IAS signal available in frequency domain. Nevertheless the DC tachometers are frequently used in industrial applications with real-time close-loop control -reported by (Khalil and El-Bardini, 2011; Barbosa *et al*., 2010; Altintas *et al*., 2011)- essentially because the input information related to IAS (delivered by tachometer) is a continuous-time signal (no significant time-delay in signal processing). A proportional-integrative (PI) control law in the regulator is appropriate to remove the electrical noise.

Despite the simplicity of DC tachometers there are some supplementary disadvantages:

– it uses permanent magnets inside: this means that the properties of these magnets may change over the time (ageing, accidental demagnetization, etc.); a periodic calibration is necessary.

– the tachometer introduces friction in the mechanical system where are placed (because of brushes and bearings); a brushless tachometer (more expensive) is sometime desirable.

– if the voltage generated by tachometer is acquired with an equipment having low electrical impedance then a reaction torque (proportional with the electrical resistance) is generated, a supplementary mechanical loading occurs when the tachometer shaft rotates.

Shah-Mohammadi-Azar *et al*. (2013) propose a capacitive sensor and a fully differential capacitance sensing method for angular position and IAS measurement. Nevertheless the relationship between output voltage signal and IAS is not linear.

Fabian and Brasseur (1998) proposes also a capacitive sensor for IAS measurement (with a relative error of 4%) using a passive rotary electrode placed between two fixed electrical active electrodes.

The older methods of speed measurement on vehicles use a drag-cup tachometer (Morris and Langari, 2016) also known as speedometer. A small permanent magnet (attached to the shaft whose IAS must be measured) rotates inside a cup made by aluminium (or copper). This cup is attached to an internal shaft which carries an angular pointer. When the magnet rotates, a variable magnetic field is created so eddy-currents occur in the cup. The interaction between currents and magnetic field creates an action torque transmitted to the internal shaft. If a reaction torque is generated on this shaft (*e.g.* with a flat spiral torsion spring) then the angle of rotary motion is proportional with IAS of permanent magnet (and with reaction torque as well). This kind of interaction occurs also in eddy-currents braking systems (Karakok *et al.*, 2016). A future approach on this topic will be probably achieved: the indirectly measurement of IAS by measuring the reaction torque or the angular displacement of internal shaft. It is expected that the dynamics of this measurement system will be a difficult item.

A resolver used to describe the angular position of a rotary shaft is supposed to produce also a continuous-time signal, according with Fig. 5. An IAS signal can be produced by numerical differentiation of angular position signal, according with (Sarma *et al*., 2008; Bourogaoui *et al*., 2016) and Fig. 6.

The advantage of these approaches is that the resolution of $\overline{\omega}$ description is practically infinite (the only limit is given by the resolution of the numerical description of the analog IAS signal deliver by sensors).

2.3. Indirect (Encoder-less) IAS Measurement Signal Techniques

The rotary shafts produce vibrations (if they are unbalanced bodies). These vibrations are transmitted to the structure where the shafts are placed (*e.g*. inside a gearbox). This structure moves (vibrates) generally as a mixture of vibratory motions (different frequencies and amplitudes) also induced by different parts (shafts, gears, bearings, etc.) or actuating equipments (*e.g*. driving motors). A vibration sensor converts these vibrations in an electrical signal. Some signal processing techniques (*e.g*. FFT analysis) are available for stationary regimes in order to extract and to describe the vibration produced by a certain shaft or a different rotary part (Hinbin and Ding, 2013). The frequency f_i of this vibration describes the average IAS value per rotation (as $\overline{\omega}_i=2\cdot\pi\cdot f_i$).

Some researchers report scientific results on this topic, as follows:

– Feng *et al*. (2016) proposes an order spectrum analysis method of vibration generated by a planetary gearbox based on iterative generalized demodulation for non-stationary complex multi-component signal analysis.

– Leclère *et al*. (2016) proposes a technique to detect the IAS of a wind turbine from a gearbox accelerometer signal (using probability density functions of the instantaneous spectra of the signal).

– Lin and Ding (2013) obtains the IAS signal of an engine (working in steady-state regimes) using the lowest frequency component of a vibration signal in frequency domain, and uses a discrete spectrum correction technique to improve the precision. There is a big advantage of this technique: it is useful during current operations of inspection and maintenance of vehicles (a simple vibration sensor replaces the function of an encoder, difficult to install).

– Urbanek *et al.* (2013) proposes a technique of vibration signal processing based on principles of phase demodulation and joint time-frequency analysis in order to obtain the IAS evolution of a selected shaft inside a rotating machine. The authors claim that this technique is available even the machine turns in non-stationary regimes.

– Ravaglioli *et al.* (2015) proposes a methodology for instantaneous rotational speed control on a turbocharger (Common-Rail Diesel fuel injection system) through a proper processing of the signal delivered by an accelerometer.

– Hong *et al.* (2017) proposes a technique of IAS indirect measurement useful for a 750-kW planetary wind turbine gearbox diagnosis (gear failure detection by vibration signal spectral analysis) based on resampling of signal produced by an accelerometer.

An interesting approach is proposed by (Ottewill and Orkisz, 2013). They shows that it is possible to perform IAS monitoring using solely the electric current absorbed by an AC voltage induction motor used to drive a rotating machinery such as a gearbox. Some of our previous researches prove that is better to use for this purpose the evolution of the active electric power absorbed by motor (Horodincă, 2010; Horodincă, 2016). The FFT of the AC electric current contains a dominant peak on the frequency of the current. In the FFT of active electric power this peak disappears.

A new approach for a future research can be defined here (as a part of a future research): the vibration signal acquired with an accelerometer can be numerically treated with an adjustable frequency multiple narrow band pass filter in order to obtain only the component induced by rotational motion of a certain shaft. The first central frequency of this multiple band pass filter is the rotational frequency (as fundamental frequency). All other central frequencies of this filter are harmonics of this fundamental.

3. IAS Signal Processing

3.1. IAS Signal Generation in Discrete-time Signal Techniques

The most common profile of the signal delivered by an IAS sensor working in discrete-time technique is a rectangular signal (*e.g.* the signal s_1 in Fig. 1). This signal is delivered directly by sensor (*e.g.* an incremental encoder) or results by pulse shaping (a periodical signal is electronically converted in a rectangular signal).

Let's assume the using of an encoder which generates N pulses per rotation. The simplest method (Li *et al*., 2005) to calculate the average value of IAS is to count (by electronically means) how many pulses (n_p) are generated in a certain period of time (t). The average value of IAS is given by:

$$\bar{\omega} = n_p \cdot \frac{2 \cdot \pi}{N \cdot t} \tag{3}$$

This method produces high measurement accuracy at high IAS but has the disadvantage that produces a low value of sampling ratio ($R = t^{-1}$) and a systematic measurement error of $\varepsilon_\omega = \pm \pi/N \cdot t$ (if the duty cycle of the rectangular signal is 50%).

An interesting way to find the IAS value is proposed by (Arabaci and Bilgin, 2012). The rectangular signal is converted from time to frequency domain using Fast Fourier Transform. The frequency f of the peak with the highest amplitude (the highest power spectral density) is involved in average IAS calculus as: $\overline{\omega}_i=2\cdot\pi\cdot f/N$.

The highest sampling ratio $R=n\cdot N$ (with n the frequency of rotation) of average IAS description can be obtained according with the signal processing method depicted in Fig. 1, also known impulse timing (or elapsed time) method according with (Leclère *et al.*, 2016; André *et al.*, 2014; Renaudin *et al.*, 2010; Li *et al.*, 2005).

Suppose that a rectangular impulse (as voltage variation low-high-low levels, or a variation between 0-1-0 logical levels) that lasts t_{s1} is generated by the encoder (as a part of a rectangular signal s_1 having a duty cycle of 50%). This impulse is ideally depicted on signal s_1 in Fig. 1 (with vertically rising and falling edges). During this time the rotary part of the encoder is suppose to rotates with a known angle $\Delta\varphi=2\cdot\pi/2\cdot N$.

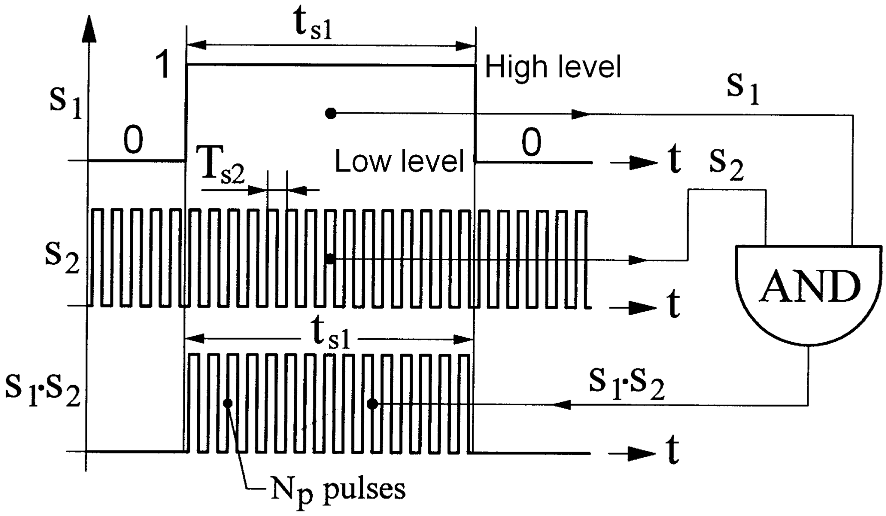

Fig. 1 – Impulse timing by elapsed time method (I).

The average IAS value during this impulse is given by:

$$\overline{\omega_{s1}}=\frac{\Delta\varphi}{t_{s1}}=\frac{\pi}{N\cdot t_{s1}} \quad (4)$$

The measurement of IAS is conditioned by the measurement of t_{s1} duration. Assume that s_2 is a clock signal with period T_{s2} (a high frequency rectangular signal -usually tens or hundreds of MHz- having a 50% duty cycle).

The signals s_1 and s_2 are applied to the two inputs of a logical AND gate which produces at output the logical conjunction of signals written as $s_1 \cdot s_2$ (or $s_1 \wedge s_2$ as well), depicted in Fig. 1 as a pulse train with $N_p \approx t_{s1}/T_{s2}$ pulses (the gate is open for signal s_2 only as long as the signal s_1 has high level).

The duration t_{s1} can be written as $t_{s1} \approx N_p \cdot T_{s2}$ with a systematic measurement error of $\varepsilon_t = \pm T_{s2}$. A higher frequency of signal s_2 (or a smaller period T_{s2} as well) generates a smaller measurement error.

In these conditions the average IAS from Eq. (4) becomes:

$$\overline{\omega_{s1}} \approx \frac{\pi}{N \cdot N_p \cdot T_{s2}} \pm \frac{\pi}{N \cdot T_{s2}} \qquad (5)$$

The last term from Eq. (5) describes the systematic measurement error ε_ω of average IAS.

The sampling ratio becomes double ($R=2 \cdot n \cdot N$) if the impulses generated by the transition between 1-0-1 levels from signal s_1 are processed in the same manner. All the previous considerations are available (including Fig. 1), except the necessity to use an inverter (a NOT logical gate), to process the signal s_1 before entering in the AND logical gate. In this way a 1-0-1 impulse become a 0-1-0 impulse.

This method produces high measurement accuracy at low IAS value. However it needs a computer assisted counter to count and read the number of pulses N_p and to make the calculus from Eq. (5).

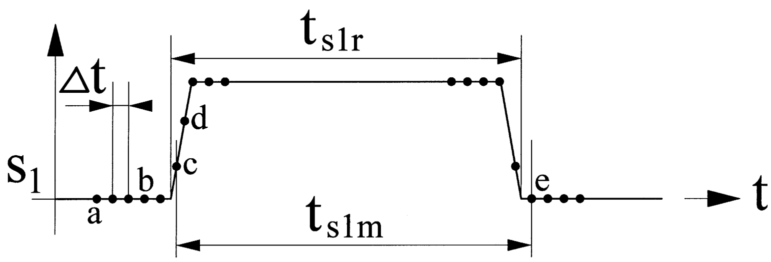

Fig. 2 – Impulse timing by elapsed time method (II).

According to Fig. 2 a simpler (but less precise) method uses the analog to digital conversion of signal s_1 (which is a common procedure in data processing) and computer aided detection for the duration of the impulse.

Here is considered the real shape of the edges, suppose to be linear and tilted slightly from vertical. The digital signal is described by distinct samples (depicted as black filled circles on Fig. 2), each sample having a precise determination by an abscissa (a time) and an ordinate (a voltage). The distance measured on abscissa between two consecutive samples is Δt (the sampling time).

The real duration of the impulse (here t_{s1r}) is detected as $t_{s1m} \approx t_{s1r} = t_e – t_c$, with t_c the time of first sample on rising edge (as the abscissa of point *c*) and t_e the time of first sample after falling edge (as the abscissa of point *e*). The measurement error occurs because the point's *c* and *e* are not placed exactly at the start and at the end of rising and falling edge.

However -as a future approach in our work- there is a way to improve a lot the accuracy of measurement (practically with $t_{s1m} = t_{s1r}$) based on the detection of the abscissa of exact point of start (end) of the rising (falling) edge, depicted as t_{sre} (t_{efe}). Here t_{sre} is the abscissa of intersection point between two lines. Each line is mathematically defined by two points *a*, *b* and *c*, *d* respectively. Similarly can be detected the time t_{efe}.

All these last considerations are available if the sampling time of the signal is small enough ($t_{s1r} >> \Delta t$). A part of these reflections will be used later in our researches to elaborate a precise detection technique of the period for any periodic signal (and particularly a harmonic signal).

3.2. IAS Signal Generation in Continuous-time Signal Techniques

The major parts of these techniques (*e.g.* DC tachometers, drag-cup tachometers etc.) don't need special procedures to obtain the IAS signal.

One of the most promising methods of measurement of IAS in continuous-time technique uses an AC resolver. Because we intend to explore this sensor in a future activity, we will focus here especially on this topic.

According to Fig. 3 and (Bourogaoui *et al.*, 2016) a resolver consists of a rotary coil RC supplied by an AC voltage $V_{IN}=V\cdot\sin(2\cdot\pi\cdot f\cdot t)$, placed near two fixed coils FC1 and FC2 having perpendicular axles (90^{o} between).

The rotary coil RC (electrically supplied via a rotary transformer) is attached inside the resolver to a shaft. This shaft is coupled to the rotary part whose rotation angle (θ) should be measured. The fixed coils are placed inside the resolver so that the rotary coil induces an AC voltage in fixed coils having the instantaneous values: $V_{OUT1}=V\cdot\underline{\mathbf{sin}}(\theta)\cdot\sin(2\cdot\pi\cdot f\cdot t)$ and $V_{OUT2}=V\cdot\underline{\mathbf{cos}}(\theta)\cdot\sin(2\cdot\pi\cdot f\cdot t)$. Here θ is the angle of RC axis measured against the axis of FC2. The ratio of the two instantaneous voltages produces the rotation angle as it follows:

$$\frac{V_{OUT1}(t)}{V_{OUT2}(t)}=\frac{V\cdot\sin(\theta)\cdot\sin(2\cdot\pi\cdot \mathrm{f}\cdot \mathrm{t})}{V\cdot\cos(\theta)\cdot\sin(2\cdot\pi\cdot \mathrm{f}\cdot \mathrm{t})}=\frac{\sin(\theta)}{\cos(\theta)}=\tan(\theta) \quad (6)$$

$$\theta(t)=\arctan\left[\frac{V_{OUT1}(t)}{V_{OUT2}(t)}\right] \quad (7)$$

A simple simultaneously measurement of the voltages V_{OUT1} and V_{OUT2}

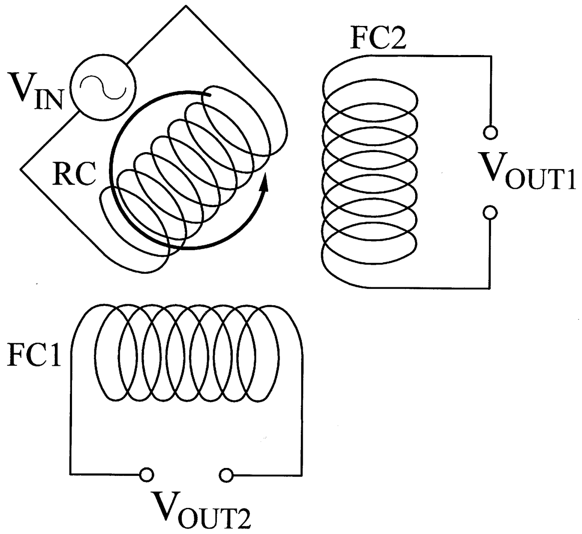

Fig. 3 – A conceptual description of a resolver.

and the calculus from Eq. (7) produces the value of the angle $\theta(t)$. A computer assisted data acquisition and calculus system based on a two channels analog-digital converter is necessary. The sampling ratio of $\theta(t)$ is the same with the sampling ratio of the voltages $V_{OUT1}(t)$ and $V_{OUT2}(t)$. Assuming an evolution of rotation angle θ related by time written as $\theta=\theta(t)$, then the IAS of rotary part is defined as numerical differentiation of this angle (related to time). By numerical differentiation two arbitrary successive values of the rotation angle $\theta_i=\theta_i(t_i)$ and $\theta_{i+1}=\theta_{i+1}(t_{i+1})$ determine the current value of the average IAS $\omega_k(t_k)$ as:

$$\overline{\omega_k} \approx \frac{\Delta\theta}{\Delta t} = \frac{\theta_{i+1} - \theta_i}{t_{i+1} - t_i} \tag{8}$$

If there are n_θ samples which describes the evolution of θ than $i=1\div n_\theta-1$. There are $n_\theta-1$ samples of $\overline{\omega}$, so $k=1\div n_\theta-1$. Here above $t_k=(t_{i+1}+t_i)/2$.

Similarly, by numerical differentiation of IAS, the average instantaneous angular acceleration (IAA) $a_j(t_j)$ can be defined (Horodincă, 2016) as:

$$\overline{a_j} \approx \frac{\Delta\omega}{\Delta t} = \frac{\omega_{k+1} - \omega_i}{t_{k+1} - t_k} \tag{9}$$

Here $j=1\div n_\theta - 2$ (there are $n_\theta - 2$ samples of average IAA) and $t_j=(t_{k+1}+t_k)/2$.

Similarly the first and second derivative of IAA (also known in physics as *jerk* and *jounce*) can be defined.

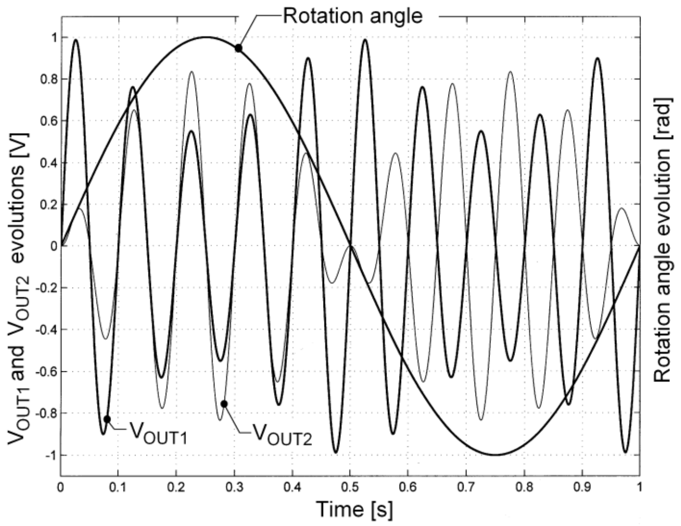

Fig. 4 – Computer aided simulation of the signals generated by a resolver: ($V_{OUT1}(t)$, $V_{OUT2}(t)$) and the calculated rotation angle $\theta(t)$ (from simulated signals).

This approach is also available for calculus of IAS using any other different technique of rotary angle measurement, *e.g.* using a capacitive sensor (Shah-Mohammadi-Azar *et al.*, 2013; Fabian and Brasseur, 1998).

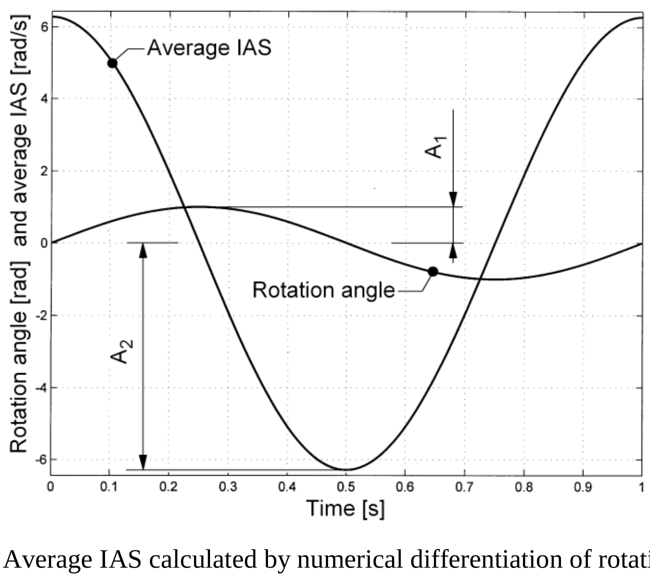

Fig. 5 – Average IAS calculated by numerical differentiation of rotation angle.

In order to prove the accessibility of this technique, a numerical simulation of the evolutions of the voltages $V_{OUT1}=1\cdot\sin(\theta)\cdot\sin(2\cdot\pi\cdot 10\cdot t)$ and $V_{OUT2}=1\cdot\cos(\theta)\cdot\sin(2\cdot\pi\cdot 10\cdot t)$ for an harmonic evolution (1 Hz frequency) of rotation angle $\theta(t)=1\cdot\sin(2\cdot\pi\cdot 1\cdot t)$ -and $V_{IN}=1\cdot\sin(2\cdot\pi\cdot 10\cdot t)$- was done and depicted in Fig. 4 (for t=0÷1 s and a sampling time of Δt=1 μs).The rotary coil RC is supplied with an AC voltage with V=1 V amplitude and f=10 Hz frequency. The evolution of rotation angle $\theta=\theta(t)$ depicted in Fig. 4 is established by calculus, from simulated values $V_{OUT1}(t)$ and $V_{OUT2}(t)$, using Eq. (7). We should mention that a real resolver uses a higher amplitude V (up to ten volts) and frequency f (usually a few hundreds of Hz) of supplying voltage V_{IN}.

Fig. 5 shows the evolution of average IAS $\overline{\omega}(t)$ obtained by numerical differentiation of rotation angle $\theta(t)$ -already introduced in Eq. (8)- as it was depicted in Fig. 4. There are two arguments which prove a correct approach for average IAS calculus:

– the IAS evolution $\overline{\omega}(t)$ is in quadrature with rotation angle $\theta(t)$ (with 90^o shift of phase between);

– the ratio A_2/A_1 between the amplitude of IAS ($A_2=2\cdot\pi$ on Fig. 5) and the amplitude of rotation angle (A_1=1 on Fig. 5) is exactly the angular velocity of rotation angle ($2\cdot\pi\cdot 1$) from $\theta(t)=1\cdot\sin(2\cdot\pi\cdot 1\cdot t)$).

These previously simulation results will be used in our future experimental researches on IAS evolution.

Because the domain of $\theta(t)$ evolution is frequently very big (*e.g.* hundreds of completely rotations) the output value of $\theta(t)$ from Eq. (7) should be systematically rectified for the reason that the function *arctan* produce results only in the domain $-\pi/2 \div \pi/2$. We already solved this issue.

3.3. IAS Signal Filtering

The IAS signal is often a mixture of periodic and non-periodic components, sometimes not related with the IAS (as electrical noise). In order to extract the evolution of a desired part of this signal the most common technique is the numerical filtering.

The major part of the periodic phenomena inside a rotary machine mirrored in the evolution of IAS has low frequency. A low-pass numerical filter is suitable to extract the influence of these phenomena on IAS evolution. The simplest way to introduce this filtering technique is the so called moving average filter (http://www.analog.com/media/en/technical-documentation/dsp-book/dsp_book_Ch15.pdf), depicted in mathematical form as:

$$\overline{\omega_{out}^{1}}(j)=\frac{1}{p}\cdot\sum_{i=j-p+1}^{i=p}\overline{\omega_{in}}(i) \qquad (10)$$

The current sample at the filter output $\overline{\omega_{out}^{1}}(j)$ is an arithmetic average of p previous samples $\overline{\omega_{in}}(i)$ applied at the filter input. Here p is the filter parameter. A numerical simulation of the filter characteristic (the evolution of ratio between output and input signal amplitude versus frequency) prove that if the IAS signal has a sampling time of Δt_{ω} (or a sampling ratio of $1/\Delta t_{\omega}$ as well) all the components having the frequencies: $1/p\cdot\Delta t_{\omega}$, $2/p\cdot\Delta t_{\omega}$ ......$p/p\cdot\Delta t_{\omega}$ are completely removed (see the points A, B, C and D on Figure 6). The amplitudes of all other components are only attenuated. It is possible to completely remove all the components of IAS signal having the frequency bigger than $1/p\cdot\Delta t_{\omega}$ (also called transition frequency) if a multiple, repetitive, moving average filter is used. This filter is mathematical described by:

$$\overline{\omega_{out}^{u}}(j) = \frac{1}{p-k} \cdot \sum_{i=j-p-k+1}^{i=p-k} \overline{\omega_{out}^{u-1}}(i) \qquad \text{with } k=s,\ 2\cdot s,\ 4\cdot s, \ldots\ldots p-2 \qquad (11)$$

The current sample at the filter output $\overline{\omega_{out}^{u}}(j)$ is an arithmetic average of p-k previous samples $\overline{\omega_{out}^{u-1}}(i)$ applied at the filter input, $\overline{\omega_{out}^{u-1}}(i)$ being also a result of a previous filtering.

Fig. 6 present the filter characteristics, also called transmissibility's versus frequency evolutions (single filtering, multiple filtering and ideal filtering).

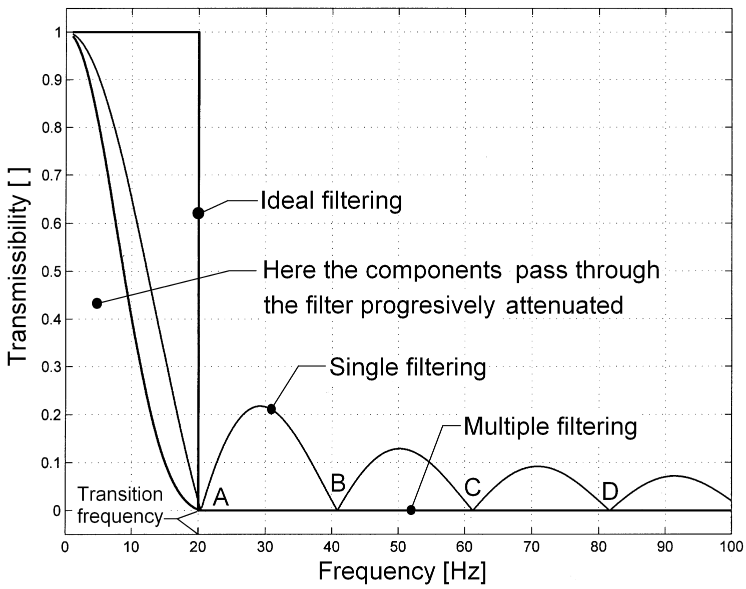

Fig. 6 – The transmissibility of a moving average low-pass filter
(with single, multiple and ideal filtering).

The efficiency of multiple average filter technique was proved in an experimental IAS signal processing (shown here only as a preliminary result). In a lathe chuck an AC tachometer (Horodincă, 2016) was placed (a stepper motor used as generator, with N=100) in order to describe the IAS evolution of the main shaft throughout a simple experiment: during the rotation motion (with 1043.3 rpm., average value) three trains of positive impulse loading (torque) was applied on the main shaft by manually braking. Each train contains ten positive impulses (not strictly repetitive) with 2.7 Hz frequency (approximately value).

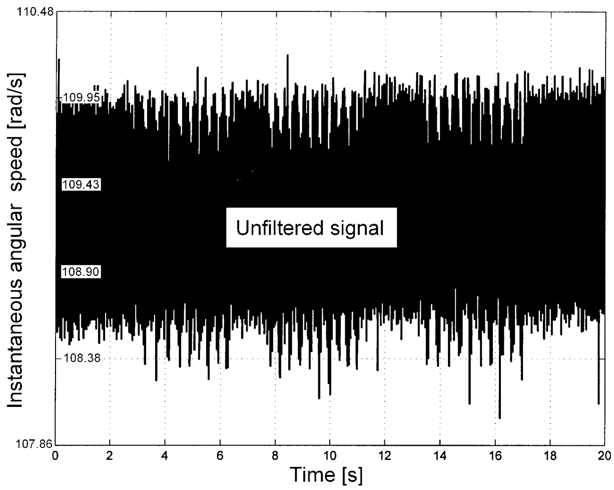

Fig. 7 – An evolution of IAS signal. The expected component of IAS is hidden by plenty of variable components.

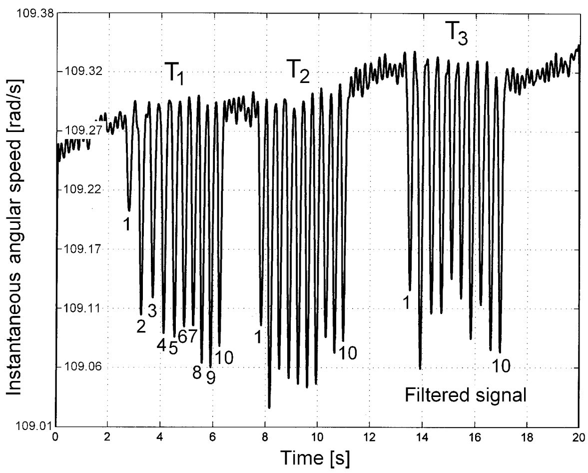

Fig. 8 – The IAS evolution given in Fig. 7 after multiple numerical moving average filtering. The expected component of IAS is clearly described.

Fig. 7 describes the evolution of IAS signal during this experiment (with 2.62 rad/s as y-axis range), with a sampling time of Δt=575.06 μs average value. It is expected that the IAS should mirror this variable loading, but unfortunately this signal contains too many variable components which hide this influence.

In order to eliminate these components a multiple moving average filter was used, according with Eq. (11), with p=140 and s=20. The result of filtering is shown in Fig. 8 (with only 0.37 rad/s as y-axis range). The mechanical loading during experiment is now very well described in the IAS evolution; here occurs clearly three trains of impulses (T_1, T_2 and T_3) each one with ten negative impulses. Because of the mechanical characteristics of the electrical motor (the IAS decreases when the loading increases) a positive impulse of loading generates a negative impulse of speed.

The efficiency of this filtering method was proved. An important remark should be formulated: according with the simulation of transmissibility filter evolution before the transition frequency (in the point A in Fig. 6) the amplitudes of the impulses from Fig. 8 is just 76.03% from real amplitudes. A multiplication factor of 1/0.7603 should be used for the ordinates in Fig. 8 in order to obtain the real amplitudes. Also is expected that the filtered signal is phase shifted relative to the real signal.

3.4. Fast Fourier Transform of the IAS Signal

An appropriate technique of IAS measurement and a stationary regime of the rotary system allow the analysis of angular speed in frequency domain using the fast Fourier transform (FFT) according with (Li and Zhang, 2017; André *et al*., 2014; Arabaci and Bilgin, 2012; Renaudin *et al*., 2010).

A variable (periodical) mechanical loading (torque) induced by malfunction of a rotary system's component (*e.g.* a shaft rotating with a frequency of rotation f_s) certainly introduces a variable component in IAS evolution. This component should be observable at least as a peak (having the frequency f_s) in the power spectral density of IAS signal obtained using FFT. Some supplementary peaks harmonically correlated with this peak should be also observable. The frequencies and amplitudes of all these peaks characterize the malfunction of system's component, useful in a diagnosis procedure.

In order to prove the availability of this approach, a preliminary study shows that almost all variable components from the IAS signal depicted in Fig. 7 are strictly related with phenomena from the lathe's gearbox. The evolution from Fig. 7 is converted in frequency domain as shown in Fig. 9 (a partial view, from 0 to 40 Hz). The conversion was done by FFT, with IAS measurement units for the amplitudes of the signal's components, on y-axis (http://www.mathworks. com/help/matlab/ref/fft.html).

Despite the fact that the signal is numerically described in time domain with a slightly variable sampling time (which depends by number of samples per rotation N and the IAS value) and it describes a variable process with a train of pulses (T_1, T_2 and T_3), the evolution from Fig. 9 indicates a low level of measurement noise. The majority of the components of the signal occur periodically, each one having a peak on Fig. 9. Some of them -as fundamental frequencies- are certainly related with the behaviour of rotating parts of the gearbox (*e.g.* A -a first driving belt, B -a second driving belt, C and E -internal shafts, D -the main shaft) some others (marked with $X_1 \div X_{11}$) are harmonics of the fundamentals (*e.g.* HB_1 and HB_2 as first and second harmonics of B, HC_1 as first harmonic of C, HD_1 as first harmonic of D) or has unknown origin (for the time being). Also the variable loading with trains of impulses T_1, T_2 and T_3 are quite well described.

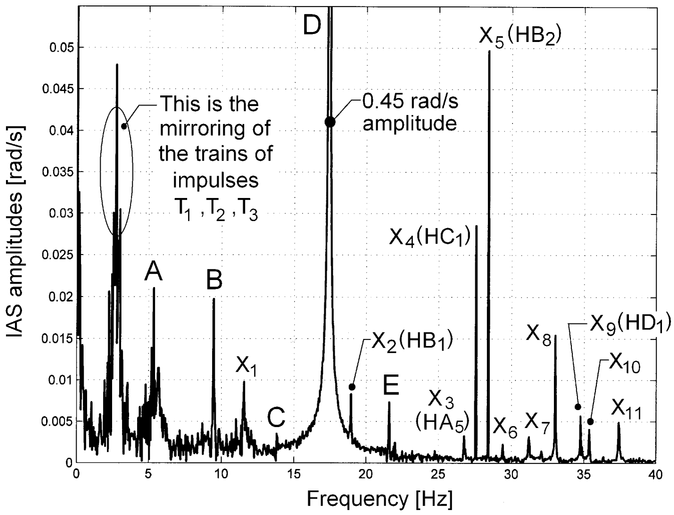

Fig. 9 – The evolution of IAS from Fig. 7 in frequency domain (obtained by FFT, with IAS measurement unit for the amplitudes of the components).

However we should mention that the FFT provides a partial characterisation of the IAS periodical components (each one described with a sum of harmonic correlated constituents). For each constituent FFT gives only the value of amplitude and the frequency (however, approximate values). For a completely characterisation is necessary also to find out the phases of this constituents (as a challenge for a future approach).

4. Conclusions

The evolution of IAS in rotary machines is an important issue in scientific research taking into account the large number of scientific papers and the reported results on this topic available in the scientific literature. The last achievements in mechatronics of IAS sensors and in computer assisted data acquisition and signal processing improve actually significantly the researcher's possibilities to discover and to fix new opportunities in this area (*e.g*. new sensors and measurement techniques, increasing the accuracy measurements, signal processing in time and frequency domain). Many issues related by mechanical dynamics of rotary parts involved in movement transmission previously partially solved using loading and vibration sensors (*e.g*. angular acceleration evolution, torsional vibrations detection, condition and fault diagnosis, etc.) are now fully approachable using IAS sensors.

Besides presenting all these new opportunities we have identified and confirmed a part of them with our own previous unpublished research results (*e.g*. the synthesis of angular displacement signal in a resolver, the numerical differentiation of angular displacement, signal processing by numerical filtering and fast Fourier transform, etc.).

Some new future research directions were defined as follows:

– The synthesis of a high precision computer assisted technique detection of the period (semi-period) for any periodical signal, especially a harmonic signal delivered by a bipolar stepper motor used as AC tachometer (N=200), useful in IAS measurement by impulses elapsed time method.

– The increasing of sampling rate in IAS description by numerically processing of the AC tachometer signals (two different methods available).

– The usage of a resolver as high sampling rate IAS sensor by numerical differentiation of the angular displacement.

– The numerical differentiation of the IAS signal in order to obtain the angular acceleration signal (and the second and third derivative of the IAS as well).

– The synthesis of a numerically adjustable frequency multiple narrow band pass filter in order to extract a periodical signal (the fundamental and harmonics correlated components) from an IAS signal. This issue is useful for diagnosis of a selected rotary part inside a machine.

– A method for precise identification of the IAS signal components (amplitude, frequency and phase) by numerical interpolation (better and more complete than by FFT).

There is an direct benefit of this last objective, if a completely identification of the periodical components from the signal depicted in Fig. 7 (except the trains of impulses T_1, T_2 and T_3) is done, then this components can be mathematically subtracted (removed) from the signal. As result, the correct mirroring of variable loading induced by the trains of impulses (generated with approximation in Fig. 8, by filtering) is obtained (with real amplitudes, no phase shifting).

Acknowledgements. We should mention that this paper valuated some experimental resources available on ENERED platform, funded by European Social Fund and Romanian Research Authority, through Developing a Research Platform for

Efficient and Sustainable Energy Grant, POSCCE-A2-02.2.1-2009-4 ID 911, implemented at “Gheoghe Asachi” Technical University of Iași, Romania.

REFERENCES

Altintas Y., Verl A., Brecher C., Uriarte L., Pritschow G., *Machine Tool Feed Drives*, Cirp. Ann. Manuf. Techn., **60**, 779-796 (2011).

André H., Girardin F., Bourdon A., Antoni J., Rémond D., *Precision of the IAS Monitoring System Based on the Elapsed Time Method in the Spectral Domain*, Mech. Syst. Signal Process., **44**, 14-30 (2014).

Aono T., Saruwatari M., Furuya J., *Estimation of Engine Torque and Cylinder Pressure Index Based on Crankshaft Rotation Measurement*, 7th IFAC Symposium on Advances in Automotive Control the International Federation of Automatic Control September 4-7, Tokyo, Japan (2013).

Arabaci H., Bilgin O., *A Novel Motor Speed Calculation Method Using Square Wave Speed Sensor Signals via fast Fourier Transform*, Turk. J. Electr. Eng. Co Journal, **20**, *1*, 1090-1099 (2012).

Barbosa R.S., Machado J.A.T., Jesus I.S., *Effect of Fractional Orders in the Velocity Control of a Servo System*, Comput. Math. Appl., **59**, 1679-1686 (2010).

Bellini A., Bifaretti S., *A Digital Filter for Speed Noise Reduction in Drives Using an Electromagnetic Resolver*, Math. Comput. Simulat., **71**, 476-486 (2006).

Bourdon A., André H., Rémond D., *Introducing Angularly Periodic Disturbances in Dynamic Models of Rotating Systems under Non-Stationary Conditions*, Mech. Syst. Signal Process., **44**, 60-71 (2014).

Bourogaoui M., Sethom H.B.A., Belkhodja I.S., *Speed/Position Sensor Fault Tolerant Control in Adjustable Speed Drives – A Review*, ISA Transactions, **64**, 269-284 (2016).

Diamond D.H., Heyns P.S., Oberholster A.J., *Online Shaft Encoder Geometry Compensation for Arbitrary Shaft Speed Profiles Using Bayesian Regression*, Mech. Syst. Signal Process., **81**, 402-418 (2016).

Espadafor F.J.J., Villanueva J.A.B., Guerrero D.P., García M.T., Trujillo E.C., Vacas F.F., *Measurement and Analysis of Instantaneous Torque and Angular Velocity Variations of a Low Speed Two Stroke Diesel Engine*, Mech. Syst. Signal Process., **49**, 135-153 (2014).

Fabian T., Brasseur G., *A Robust Capacitive Angular Speed Sensor*, IEEE Transactions on Instrumentation and Measurement Conf. IMTC, Ottawa, Canada, May 19–21, 1997, 1267-1272 (1998).

Feng Z., Chen X., Liang M., *Joint Envelope and Frequency Order Spectrum Analysis Based on Iterative Generalized Demodulation for Planetary Gearbox Fault Diagnosis under Nonstationary Conditions*, Mech. Syst. Signal Process., **76-77**, 242-264 (2016).

Girardin F., Rémond D., Rigal J.F., *Tool Wear Detection in Milling an Original Approach with an on Dedicated Sensor*, Mech. Syst. Signal Process., **24**,1907-1920 (2010).

Gomez J.L., Bourdon A., André H., Rémond D., *Modelling Deep Groove Ball Bearing Localized Defects Inducing Instantaneous Angular Speed Variations*, Tribol. Int., **98**, 270-281 (2016).

Gu F., Yesilyurt I., Li Y., Harris G., Ball A., *An Investigation of the Effects of Measurement Noise in the Use of Instantaneous Angular Speed for Machine Diagnosis*, Mech. Syst. Signal Process., **20**, 1444-1460 (2006).

Gubran A.A., Sinha J.K., *Shaft Instantaneous Angular Speed for Blade Vibration in Rotating Machine*, Mech. Syst. Signal Process., **44**, 47-59 (2014).

Guo Y., Li W., Yu S., Han X., Yuan Y., Wang Z., Ma X., *Diesel Engine Torsional Vibration Control Coupling with Speed Control System*, Mech. Syst. Signal Process., **94**, 1-13 (2017).

Hinbin L., Ding K., *A New Method for Measuring Engine Rotational Speed Based on the Vibration and Discrete Spectrum Correction Technique*, Measurement, **46**, 2056-2064 (2013).

Hong L., Yongzhi Q., Dhupia J.S., Sheng S., Yuegang T., Zude Z., *A Novel Vibration-Based Fault Diagnostic Algorithm for Gearboxes under Speed Fluctuations without Rotational Speed Measurement*, Mech. Syst. Signal Process., **94**, 14-32 (2017).

Horodinca M., *Some New Resources on Computer Assisted Experimental Research of the Absorbed Electric Power in Manufacturing Systems*, Bul. Inst. Polit. Iaşi, **LVI (LX)**, *2*, s. Constr. Maş., 125-133 (2010).

Horodinca M., *The Monitoring of Transient Regimes on Machine Tools Based on Speed, Acceleration and Active Electric Power Absorbed by Motors*, Proc. 4th Internat. Conf. on Modern Techn. in Ind. Eng., ModTech 2016, IOP Conf. Series: Mat. Sci. and Eng., **145**, *2*, 1-8 (2016).

Karakoc K., Suleman A, Park E.J., *Analytical Modeling of Eddy Current Brakes with the Application of Time Varying Magnetic Fields*, Appl. Math. Model., **40**, 1168-1179 (2016).

Khalil H.M., El-Bardini M., *Implementation of Speed Controller for Rotary Hydraulic Motor Based on LS-SVM*, Expert. Syst. Appl., **38**, 14249-14256 (2011).

Kim D., Park J., *Application of Adaptive Control to the Fluctuation of Engine Speed at Idle*, Inform. Science, **177**, 3341-3355 (2007).

Lamraoui M., Thomas M., ElBadaoui M., Girardin F., *Indicators for Monitoring Chatter in Milling Based on Instantaneous Angular Speeds*, Mechanical Systems and Signal Processing, **44**, 72-85 (2014).

Leclère Q., Girardin F., Rémond D., *An Analysis of Instantaneous Angular Speed Measurement Errors*, Surveillance 7 International Conference, Oct 2013, CHARTRES, France, 1-11, 201 (2013).

Leclère Q., Hugo A., Antoni J., *A Multi-Order Probabilistic Approach for Instantaneous Angular Speed Tracking Debriefing of the CMMNO'14 Diagnosis Contest*, Mech. Syst. Signal Process., **81**, 375-386 (2016).

Li B., Zhang X., *A New Strategy of Instantaneous Angular Speed Extraction and its Application to Multistage Gearbox Fault Diagnosis*, J. Sound. Vib., **396**, 340-355 (2017).

Li B., Zhang X., Wu J., *New Procedure for Gear Fault Detection and Diagnosis Using Instantaneous Angular Speed*, Mech. Syst. Signal Process., **85**, 415-428 (2017).

Li Y., Gua F., Harris G., Ball A., Bennett N., Travis K., *The Measurement of Instantaneous Angular Speed*, Mech. Syst. Signal Process., **19,** *4*, 786-805 (2005).

Lin H., Ding K., *A New Method for Measuring Engine Rotational Speed Based on the Vibration and Discrete Spectrum Correction Technique*, Measurement, **46,** 2056-2064 (2013).

Morris A.S., Langari R., *Measurement and Instrumentation (Second Edition), Chapter 20: Rotational Motion Transducers*, Elsevier Academic Press., 599-632 (2016).

Ottewill J.R., Orkisz M., *Condition Monitoring of Gearboxes Using Synchronously Averaged Electric Motor Signals*, Mech. Syst. Signal Process., **38**, 482-498 (2013).

Pan C., Chen L., Chen L., Jiang H., Li Z., Wang S., *Research on Motor Rotational Speed Measurement in Regenerative Braking System of Electric Vehicle*, Mech. Syst. Signal Process., **66-67**, 829-839 (2016).

Ravaglioli V., Cavina N., Cerofolini A., Corti E., Moro D., Ponti F., *Automotive Turbochargers Power Estimation Based on Speed Fluctuation Analysis*, Energy Procedia, **82,** 103-110 (2015).

Renaudin L., Bonnardot F., Musy O., Doray J.B., Rémond D., *Natural Roller Bearing Fault Detection by Angular Measurement of True Instantaneous Angular Speed*, Mech. Syst. Signal Process., **24**, 1998-2011 (2010).

Rivola A., Troncossi M., *Zebra Tape Identification for the Instantaneous Angular Speed Computation and Angular Resampling of Motorbike Valve Train Measurements*, Mech. Syst. Signal Process., **44**, 5-13 (2014).

Sarma S., Agrawal V.K., Udupa S., Parameswaran K., *Instantaneous Angular Position and Speed Measurement Using a DSP Based Resolver-to-Digital Converter*, Measurement, **41**, 788-796 (2008).

Shah-Mohammadi-Azar A., Azimloo H., Rezazadeh G., Shabani R., Tousi B., *On the Modeling of a Capacitive Angular Speed Measurement Sensor*, Measurement, **46**, 3976-3981 (2013).

Shimano S., Shiono M., Ohnishi K., *Estimation of Acceleration by an AC Tachogenerator and its Applications to Servo Control*, Electr. Eng. Jpn., **110**, *6*, 90-97 (1990).

Stander C.J., Heyns P.S., *Instantaneous Angular Speed Monitoring of Gearboxes under Non-Cyclic Stationary Load Conditions*, Mech. Syst. Signal Process., **19**, 817-835 (2005).

Urbanek J., Barszcz T., Antoni J., *A Two-Step Procedure for Estimation of Instantaneous Rotational Speed with Large Fluctuations*, Mech. Syst. Signal. Process., **38**, 96-102 (2013).

Villanueva B.J.A., Espadafor J., Cruz-Peragón F., García M.T., *A Methodology for Cracks Identification in Large Crankshafts*, Mech. Syst. Signal Process., **25**, 3168-3185 (2011).

Yu S.D., Zhang X., *A Data Processing Method for Determining Instantaneous Angular Speed and Acceleration of Crankshaft in an Aircraft Engine–Propeller System Using a Magnetic Encoder*, Mech. Syst. Signal Process., **24**, 1032-1048 (2010).

Zhao M., Jia X., Lin J., Lei Y., Lee J., *Instantaneous Speed Jitter Detection via Encoder Signal and its Application for the Diagnosis of Planetary Gearbox*, Mech. Syst. Signal Process., **98**, 16-31 (2018).

Wu Z., Bian L., Wang S., Zhang X., *An Angle Sensor Based on Magnetoelectric Effect*, Sensors and Actuators **A 262**, 108-113 (2017).

*** http://www.mathworks.com/help/matlab/ref/fft.html (accessed at 4.02.2017).
*** http://www.analog.com/media/en/technical- documentation/dspbook/dsp_book_Ch15.pdf (accessed at 4.02.2017).

CONSTITUIREA ŞI PRELUCRAREA SEMNALULUI DE DESCRIERE A VITEZEI UNGHIULARE: O SCURTĂ EVALUARE A STADIULUI ACTUAL

(Rezumat)

Lucrarea propune o scurtă prezentare a stadiului actual în legătură cu cele mai cunoscute tehnici de măsurare şi de prelucrare a semnalului de descriere a vitezei unghiulare instantanee (VUI). Se prezintă pe scurt resursele şi rezultatele deja disponibile în literatura ştiinţifică cu scopul de a identifica obiectivele cercetărilor noastre viitoare. Au fost mai întâi prezentate tehnicile principale de măsurare directă şi indirectă a VUI (cu impulsuri, cu descriere continuă, fără senzor VUI). Au fost făcute apoi unele consideraţii asupra tehnicilor cunoscute de producere a semnalului VUI (cronometrarea impulsurilor, derivarea numerică a semnalului de descriere a deplasării unghiulare) şi de prelucrare a acestora (filtrare numerică, transformata Fourier rapidă), folosind frecvent spre exemplificare rezultate ale propriilor noastre cercetări.